

Nearly Instantaneous Alternatives in Quantum Mechanics

Richard J. Micanek

Department of Physics

University of California

Santa Barbara, CA 93106-9530

micanek@vorpall.ucsb.edu

James B. Hartle

Institute for Theoretical Physics

University of California

Santa Barbara, CA 93106-9530

hartle@itp.ucsb.edu

Abstract

Usual quantum mechanics predicts probabilities for the outcomes of measurements carried out at definite moments of time. However, realistic measurements do not take place in an instant, but are extended over a period of time. The assumption of instantaneous alternatives in usual quantum mechanics is an approximation whose validity can be investigated in the generalized quantum mechanics of closed systems in which probabilities are predicted for spacetime alternatives that extend over time. In this paper we investigate how alternatives extended over time reduce to the usual instantaneous alternatives in a simple model in non-relativistic quantum mechanics. Specifically, we show how the decoherence of a particular set of spacetime alternatives becomes automatic as the time over which they extend approaches zero and estimate how large this time can be before the interference between the alternatives becomes non-negligible. These results suggest that the time scale over which coarse grainings of such quantities as the center of mass position of a massive body may be extended in time before producing significant interference is much longer than characteristic dynamical time scales.

I. INTRODUCTION

As usually formulated, quantum mechanics predicts probabilities for the outcomes of measurements carried out at definite moments of time. However, realistic measurements do not take place in an instant, but are extended over a period of time. The assumption of instantaneous measurements in usual quantum mechanics is an approximation whose validity can be investigated in the generalized quantum mechanics of closed systems where probabilities are predicted for alternatives that are extended over time. In this paper we investigate how alternatives extended over time reduce to the usual alternatives at a moment of time in a simple model in non-relativistic quantum mechanics.

There are a number of interesting discussions of ideal measurements extended over time in the quantum mechanical literature [1,2,3,4,5,6,7,8]. However, these discussions were incomplete because they did not completely specify what an “ideal” measurement consisted of nor what replaced the reduction of the state vector upon the completion of one. In the more general quantum mechanics of closed systems, in which a notion of “measurement” does not play a fundamental role, clear meaning can be given to the probabilities for spacetime alternatives which extend over time [9,10,11].

Spacetime alternatives are easily visualized in a sum-over-histories formulation of quantum mechanics. Consider for example the quantum mechanics of a single non-relativistic particle moving in one spatial dimension. The *fine-grained-histories* for this system are the particle paths $x(t)$ on a time interval, say, $[0, T]$. The most general sets of alternatives for this system are partitions of this set of fine-grained paths into sets of coarse-grained classes, c_α , where $\alpha = 1, 2, 3 \dots$. Partitions by the values of x at which paths cross a surface of time t are simple kinds of coarse grainings that correspond to the usual alternatives of position at a moment of time. Partitions into classes whose definition extends over time generalize these to spacetime alternatives. A simple example is a partition of the paths defined by whether they cross or never cross a given spacetime region with extent in time.

For each class of paths c_α , a class operator C_α may be defined by a path integral over paths in the class of the form

$$\langle x'' | C_\alpha | x' \rangle = \int_\alpha dx \exp(iS[x(\tau)]/\hbar). \quad (1)$$

Here, $S[x(\tau)]$ is the action functional for paths and the sum is over all paths in the class c_α . This class operator incorporates both unitary dynamics and a generalization of state

vector reduction in a unified way. When the system's initial state at $t = 0$ is $|\Psi\rangle$, the probability of coarse-grained alternative α is

$$p(\alpha) = \|C_\alpha|\Psi\rangle\|^2. \quad (2)$$

However, the quantum mechanics of closed systems does not predict probabilities for every set of coarse-grained alternatives that may be described, but only for those which have negligible quantum mechanical interference between the individual histories in the set. Such sets are said to decohere. Specifically, sets of histories for which consistent probabilities are predicted must satisfy a decoherence condition, which for present purposes we may take to be

$$\langle\Psi|C_{\alpha'}^\dagger \cdot C_\alpha|\Psi\rangle \sim 0, \quad \alpha' \neq \alpha. \quad (3)$$

All the required probability sum rules are satisfied by the probabilities defined by (2) as a consequence of the decoherence condition (3).

It is a straightforward calculation [10] to verify that alternatives defined at one moment of time t have class operators of the form

$$C_\alpha = e^{-iHT} P_\alpha(t) \quad (4)$$

where H is the Hamiltonian of the closed system and $\{P_\alpha(t)\}$ are a set of orthogonal Heisenberg picture projection operators. (Here, as throughout, we use units in which $\hbar = 1$.) For instance, for the alternatives that the particle is in one of a set of exclusive spatial regions $\{\Delta_\alpha\}$ the P 's are the projections onto these regions at the appropriate time. It is then immediate from (3) that decoherence is automatic for such instantaneous alternatives.

Decoherence is not automatic for alternatives that are extended over time. However as the time T over which they extend approaches zero they must become decoherent and their class operators must approach the form (4). This paper examines this approach of spacetime alternatives to instantaneous ones in a simple model. The model is described in Section II and its behavior for small T found in Sections III – V. The significance of the results is discussed in Section VI.

II. A MODEL COARSE-GRAINING

Our model concerns a non-relativistic particle of mass M , moving in one dimension, in a potential $V(x)$. The fine-grained histories for this system between times $t = 0$ and

$t = T$ are particle paths represented by single valued functions $x(t)$ on that interval. Coarse-grainings are generally partitions of these paths into sets of exclusive classes. The individual classes are called coarse-grained histories.

As a simple example of a spacetime coarse graining that extends over time, consider the region of spacetime R that lies between times $t = 0$ and $t = T$. Denote the subregions of R that lie to the left and to the right of the origin as R_l and R_r , respectively. The set of all paths between $t = 0$ and $t = T$ may be partitioned by whether they cross or do not cross the left and right regions R_l and R_r . Specifically, let c_{10} be the class of paths which remain to the left of the origin and never cross R_r , let c_{01} be the class of paths that stay to the right of the origin and never cross R_l , and let c_{11} be the class of paths that cross both regions sometime. The class c_{00} of paths that never cross either R_l or R_r is empty and we shall not discuss it further. This set of coarse-grained histories provides a simple model for investigating the limit of small temporal extension T . In that limit, the set of spacetime alternatives should approximately decohere. Further, the probability for the alternative c_{01} in which the particle is localized on the right during time T should approach the usual probability for the particle's position at $t = 0$ to be located in the region $x > 0$. There should be a similar approach of the probability for c_{10} to the usual probability that the particle is located in $x < 0$ at $t = 0$. The probability of the alternative c_{11} that the particle is in *both* regions of x should approach zero. In the following we shall show that these expectations are correct.

As we shall show in Section V, a bounded potential $V(x)$ has a negligible effect on the class operators for very short times T . We therefore begin with an investigation of their form for a free particle with $V(x) = 0$. The class operators for the model coarse-graining were calculated in [10] but we briefly review their construction here. They may all be expressed in terms of the free particle propagator for the time interval T which is

$$K_T(x'', x') = \left(\frac{\lambda}{i\pi} \right)^{\frac{1}{2}} e^{i\lambda(x'' - x')^2}, \quad (5)$$

where

$$\lambda = \frac{M}{2T} \quad (6)$$

is a parameter which becomes large as T becomes small. Consider for example the operator C_{01} corresponding to the class of paths that remain entirely to the right of $x = 0$ for the time T . Its matrix elements are given by path integrals of the form (1) over this class of paths. That path integral is the same as the path integral over *all* paths with an action

including an infinite barrier potential for $x < 0$. That is, the class operator is the ordinary quantum mechanical propagator in the presence of an infinite reflecting barrier at $x = 0$. The appropriate solution of the Schrödinger equation may be found by the method of images and is

$$\langle x''|C_{01}|x'\rangle = \theta(x'')\theta(x') [K_T(x'', x') - K_T(-x'', x')] . \quad (7)$$

Similarly,

$$\langle x''|C_{10}|x'\rangle = \theta(-x'')\theta(-x') [K_T(x'', x') - K_T(-x'', x')] . \quad (8)$$

To find the remaining class operator C_{11} note that a sum of the form (1) over *all* paths just gives the usual free particle propagator, so that

$$\sum_{\alpha} C_{\alpha} = e^{-iHT} . \quad (9)$$

Thus

$$\begin{aligned} \langle x''|C_{11}|x'\rangle &= K_T(x'', x') - \langle x''|C_{01}|x'\rangle - \langle x''|C_{10}|x'\rangle \\ &= [\theta(x'')\theta(-x') + \theta(-x'')\theta(x')] K_T(x'', x') \\ &\quad + [\theta(x'')\theta(x') + \theta(-x'')\theta(-x')] K_T(-x'', x') . \end{aligned} \quad (10)$$

With these class operators the decoherence functional for this set of coarse-grained histories may be computed in the limit of vanishing temporal T .

III. THE DECOHERENCE FUNCTIONAL

We now turn to the question of the decoherence and probabilities of the set of space-time alternatives described above as a function of the time T over which they extend. For simplicity, we assume that at $t = 0$ the particle is in a pure state represented by a wave function $\Psi(x)$. The decoherence functional defined by

$$D(c_{\alpha}, c_{\alpha'}) = \langle \Psi | C_{\alpha'}^{\dagger} C_{\alpha} | \Psi \rangle \quad (11)$$

is a convenient tool for summarizing the essential features of the quantum mechanics of closed systems that were mentioned in the Introduction. A set of spacetime alternatives decoheres when the off-diagonal elements of D are negligibly small; the probabilities of a decoherent set of alternatives are given by the diagonal elements of D .

The decoherence functional (11) may be expressed in terms of the matrix elements of the class operators found in the preceding Section by writing

$$D(c_\alpha, c_{\alpha'}) = \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dx' \int_{-\infty}^{+\infty} dx'' \Psi(x') \Psi^*(x'') \langle x | C_\alpha | x' \rangle \langle x | C_{\alpha'} | x'' \rangle^* . \quad (12)$$

Given that $c_{00} = 0$ is empty, it is clear that any element of D involving this class vanishes identically. Additionally, since $\theta(x)\theta(-x) = 0$ we have

$$D(c_{01}, c_{10}) = D^*(c_{10}, c_{01}) = 0 . \quad (13)$$

The hermiticity of the decoherence functional implies the additional relations $D(c_{01}, c_{11}) = D^*(c_{11}, c_{01})$ and $D(c_{10}, c_{11}) = D^*(c_{11}, c_{10})$, leaving only five components to compute. We now describe how to carry out the integrals in (12) for these components.

We begin with the diagonal element $D(c_{01}, c_{01})$. It follows from (11) that this element of the decoherence functional is the square of the norm of the vector $C_{01}|\Psi\rangle$. The class operator C_{01} given by (7) is the propagator for Schrödinger evolution over time T of a free particle in the presence of an infinite barrier on $x < 0$. Applied to $|\Psi\rangle$ it gives the *branch* wave function $\Psi_{01}(x) = \langle |C_{01}|\Psi \rangle$ with

$$\Psi_{01}(x, T) \equiv \int_0^\infty dx' [K_T(x, x') - K_T(x, -x')] \Psi(x') = \int_{-\infty}^{+\infty} dx' K_T(x, x') \tilde{\Psi}(x') , \quad (14)$$

where

$$\tilde{\Psi}(x) = \Psi(x) , x > 0 \quad \text{and} \quad \tilde{\Psi}(x) = -\Psi(-x), x < 0. \quad (15)$$

This is similar to evolution of the usual Schrödinger equation but with discontinuous initial data. Notice that the probability integrals are unaffected because this data resides in the Hilbert space L_2 . Thus the norm evaluated on the interval $x \geq 0$ is preserved under this evolution, so that

$$D(c_{01}, c_{01}) = \int_0^\infty dx |\Psi(x)|^2 \equiv p_+ \quad (16)$$

and is *independent* of the time interval T . Similarly,

$$D(c_{10}, c_{10}) = \int_{-\infty}^0 dx |\Psi(x)|^2 \equiv p_- . \quad (17)$$

When the set of spacetime alternatives decoheres, the diagonal element $D(c_{01}, c_{01})$ is the probability that the particle remains at positive values of x throughout the time interval T . For this model this is the same as the probability that the particle is in this range of x at $t = 0$.

The remaining elements of the decoherence functional, which we expect to vanish in the limit of vanishing T , may be evaluated as follows: Consider as a typical example the interference term $D(c_{01}, c_{11})$. Using the results of Section II for the matrix elements of the class operators, the integrations in (12) can be rearranged to give

$$D(c_{01}, c_{11}) = \int_0^\infty dx \int_0^\infty dx' \int_0^\infty dx'' K_T^*(x, -x'') [K_T(x, x') - K_T(x, -x')] \Psi_S^*(x'') \Psi(x') \quad (18)$$

where

$$\Psi_S(x) = \Psi(x) + \Psi(-x) . \quad (19)$$

It is convenient to rescale the integration variables by writing $x = y/\lambda^{1/2}$ with similar rescalings for x' and x'' . Then, using the explicit form (5) for the free particle propagator $K_T(x'', x')$, we have

$$D(c_{01}, c_{11}) = \frac{1}{\pi \lambda^{1/2}} \int_0^\infty dy \int_0^\infty dy' \int_0^\infty dy'' e^{-i(y+y'')^2} [e^{i(y-y')^2} - e^{i(y+y')^2}] \times \Psi_S^*\left(\frac{y''}{\lambda^{1/2}}\right) \Psi\left(\frac{y'}{\lambda^{1/2}}\right) . \quad (20)$$

From this expression it is clear that only the behavior of $\Psi(x)$ near $x = 0$ will contribute in the limit of large λ and small T . Inserting a factor of $e^{-\epsilon(y'^2+y''^2)}$ to ensure the convergence of the integral, the expression has the following small T asymptotic form for finite ϵ :

$$D_\epsilon(c_{01}, c_{11}) \sim \frac{4}{i\pi \lambda^{1/2}} |\Psi(0)|^2 \int_0^\infty dy' e^{-\xi^* y'^2} \int_0^\infty dy'' e^{-\xi y''^2} \int_0^\infty dy e^{-2iy''y} \sin(2y'y) \quad (21)$$

where

$$\xi = \epsilon + i . \quad (22)$$

Using eqs. (3.893.1), (3.466.1) and (6.286.1) of [12], the integral in (21) may be evaluated explicitly in terms of a hypergeometric function:

$$\frac{1}{4} \sqrt{\frac{\pi}{\xi}} {}_2F_1\left(\frac{1}{2}, 1; \frac{3}{2}; \frac{2\epsilon}{\xi}\right) . \quad (23)$$

Now insert this in (21) and pass to the $\epsilon \rightarrow 0$ limit to remove the convergence factor and find the simple result:

$$D(c_{01}, c_{11}) \sim -\frac{e^{\frac{i\pi}{4}}}{\sqrt{\pi\lambda}} |\Psi(0)|^2. \quad (24)$$

No additional calculation is needed to evaluate the remaining off-diagonal component $D(c_{10}, c_{11})$. That is because, using the class operator matrix elements from Section II and changing the sign of the integration variables x , x' and x'' , one obtains an expression for $D(c_{10}, c_{11})$ which is identical to the right hand side of (18) except with $\Psi(x')$ replaced with $\Psi(-x')$. However, in the limit of small T , only the value $\Psi(0)$ is important as (21) shows. The leading orders of $D(c_{10}, c_{11})$ and $D(c_{01}, c_{11})$ therefore coincide for small T , both given by the right hand side of (24).

The remaining element of the decoherence functional is $D(c_{11}, c_{11})$. This can be evaluated using the techniques employed above for the other elements which vanish in the limit of small T . However, it is quicker simply to evaluate it from the general relation

$$\sum_{\alpha, \alpha'} D(c_\alpha, c_{\alpha'}) = 1 \quad (25)$$

which follows from (9) and (11). Either way the result is:

$$D(c_{11}, c_{11}) \sim 2\sqrt{\frac{2}{\pi\lambda}} |\Psi(0)|^2 \quad (26)$$

to leading order in λ .

Putting these results together, the small T behavior of the decoherence functional for the three nontrivial alternatives is given by

$$D(c_\alpha, c_{\alpha'}) = \begin{pmatrix} p_+ & 0 & 0 \\ 0 & p_- & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & \eta \\ 0 & 0 & \eta \\ \eta^* & \eta^* & -2[\eta + \eta^*] \end{pmatrix} + \dots \quad (27)$$

with p_\pm as in (16) and (17) and

$$\eta = -\frac{e^{\frac{i\pi}{4}}}{\sqrt{\pi\lambda}} |\Psi(0)|^2, \quad (28)$$

where row and column are taken in the order c_{01}, c_{10}, c_{11} . The following points are immediate consequences of this expression:

- (1) The off-diagonal elements of the decoherence functional vanish as $T^{1/2}$ in the limit of small T so that this set of spacetime alternatives decoheres in that limit.

(2) In the limit of vanishing T , non-vanishing diagonal elements of the decoherence functional coincide exactly with the probabilities for the particle to be on the left or the right of $x = 0$ at the moment of time $t = 0$, as expected.

(3) Unlike the case of histories which are sequences of sets of alternative projections, the diagonal elements of the decoherence functional for spacetime alternatives are not probabilities when the set of alternatives are not decoherent. They do not sum to one. (See [10] for a more general discussion.)

(4) The leading order of the interference terms vanishes if $\Psi(0) = 0$. That is consistent with the results of [11] who showed that decoherence is exact for any time interval T as long as the initial wave function $\Psi(x)$ is antisymmetric about $x = 0$ (a result which follows immediately from (18) since $\Psi_S(x)$ then vanishes).

IV. A SPECIFIC INITIAL CONDITION

An initial Gaussian wave packet provides an example in which we can explicitly evaluate the decoherence functional for the spacetime coarse graining under discussion without recourse to the limit of small T . Assume a one-dimensional Gaussian initial wave packet for our free particle of the form

$$\Psi(x) = \left(\frac{2}{\pi \ell^2} \right)^{\frac{1}{4}} e^{-x^2/\ell^2} \quad (29)$$

where ℓ is the characteristic width of the wave packet. With (29) into (12), evaluation of the relevant integrals that are simply those of the preceding Section yields

$$D(c_\alpha, c_{\alpha'}) = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & \gamma \\ 0 & 0 & \gamma \\ \gamma^* & \gamma^* & -2[\gamma + \gamma^*] \end{pmatrix} \quad (30)$$

as the desired decoherence matrix with

$$\gamma = \frac{1}{i\pi} \text{arctanh}^{-1} \left(\sqrt{\frac{2}{1 + i\ell^2\lambda}} \right) = \eta + \dots, \quad (31)$$

having put $\Psi(0) = \left(\frac{2}{\pi \ell^2} \right)^{\frac{1}{4}}$ in (28) for this special case. As expected, in the limit of large λ or small T , the results of (27) are reproduced and decoherence is achieved with equal probabilities of 1/2 for each of the non-vanishing coarse-grained alternatives.

V. INCLUDING A POTENTIAL

We now consider the effect of including a bounded potential on the above results derived for a free particle. Classically, the effect of a bounded force on the motion of a particle is proportional to the time interval over which it acts as long as that time interval is sufficiently small. The effect of a potential is thus negligible for very small time intervals. We expect a similarly negligible effect of a bounded potential on the propagation of a particle in quantum mechanics when the time of propagation becomes small (as it does in the coarse grainings under discussion in this paper). We shall now show that this is the case.

The general arguments involving the conservation of probability that led to the results (16) and (17) for the diagonal elements of the decoherence functional, $D(c_{01}, c_{01})$ and $D(c_{10}, c_{10})$, respectively, are as valid in the presence of a potential $V(x)$ as they were without. The values of these elements of D are therefore unchanged by the inclusion of a potential for any value of T .

To understand the effect of a potential on the other elements of the decoherence functional which vanish in the limit of small T , we need to examine the propagator over a time interval T . In the presence of a potential this is:

$$G_T(x'', x') = \langle x'' | e^{-i(H_0 + V)T} | x' \rangle , \quad (32)$$

where H_0 is the free particle Hamiltonian. It is not difficult to see that introducing a potential changes the construction of the class operator matrix elements in Section II only by replacing the free particle propagator $K_T(x'', x')$ with $G_T(x'', x')$. The results for the decoherence functional can then all be expressed in terms of inner products of wave functions of the form

$$\int dx' G_T(x, x') \Psi(x') . \quad (33)$$

We now find explicit expressions for the modifications induced by the potential on wave functions of this form.

The evolution operator $U(t) \equiv \exp[-i(H_0 + V)t]$ satisfies the Schrödinger equation

$$i dU(t)/dt = (H_0 + V)U(t) . \quad (34)$$

The correct solution can be written in the form

$$U(t) = e^{-iH_0 t} \left[1 - i \int_0^t dt' e^{iH_0 t'} V e^{-iH_0 t'} + \dots \right] . \quad (35)$$

Since the free particle propagator is

$$K_T(x'', x') = \langle x'' | e^{-iH_0 T} | x' \rangle , \quad (36)$$

this result can be used to write the evolution of an initial wave function $\Psi(x)$ over a time interval T in the form

$$\int_{-\infty}^{+\infty} dx' G_T(x, x') \Psi(x') = \int_{-\infty}^{+\infty} dx' K_T(x, x') \Psi_T^V(x') \quad (37)$$

where

$$\Psi_T^V(x) = [1 - i \int_0^T dt' e^{iH_0 t'} V e^{-iH_0 t'} + \dots] \Psi(x) . \quad (38)$$

The calculations of the small time behavior of the decoherence functional are thus the same as those in Section III with $\Psi_T^V(x)$ replacing $\Psi(x)$. However, since we are interested only in the leading order in small T , we may employ only the leading order in $\Psi_T^V(x)$. That just comes from the first term in the sum in (38), the higher order terms in the potential vanishing as successively higher order powers of T . The leading term is thus just $\Psi(x)$. Including a potential $V(x)$ therefore does not change the small T asymptotic form of the decoherence functional given by (27).

VI. DISCUSSION

We have investigated a very simple example of a spacetime coarse graining in the quantum mechanics of a single non-relativistic particle of mass M moving in one dimension. The three non-empty coarse-grained alternatives are whether the particle remains always to the right of $x = 0$ for a time interval T , remains always to the left of $x = 0$ for this interval, or is sometimes on the left and sometimes on the right during that time. As the example discussed in the previous Section shows, when the initial state is a wave packet of width ℓ , the characteristic time scale for the automatic decoherence of this set of alternatives is $T_{\text{decoherence}} \sim M\ell^2/\hbar$. When $T \ll T_{\text{decoherence}}$, the interference between these alternatives is negligible and the set approximately decoheres. The exact decoherence of instantaneous alternatives is thus a good approximation to the nearly exact decoherence of this type of spacetime alternative. Further, the probabilities of the instantaneous alternatives are exactly the same as those of the spacetime ones. When $T \ll T_{\text{decoherence}}$ instantaneous alternatives are an excellent approximation to this kind of spacetime alternative.

To get a feel for this scale, consider an electron localized to its Compton wavelength or a hydrogen atom sitting in its ground state. In these cases, $T_{\text{decoherence}} \sim 10^{-19}s$ and $T_{\text{decoherence}} \sim 10^{-14}s$, respectively. Thus for these systems spacetime coarse grainings can extend only over very short time scales if they are to be approximated by instantaneous alternatives. At a much larger scale, take the center of mass of a grain of dust with a diameter of about one micron and a corresponding mass on the order of $10^{-15}kg$ localized to its dimension. The resulting time scale for decoherence is then on the order of a year. For any macroscopic particle (for example with a mass of one gram and a size on the order of a centimeter) $T_{\text{decoherence}}$ is enormously greater than the age of the universe. For such systems the time scale $T_{\text{decoherence}}$ over which this kind of spacetime alternatives may extend while still automatically decohering is much longer than characteristic dynamical time scales $T_{\text{dynamical}}$.

These results suggest that, for quantities such as the center of mass position of a body characterized by typical macroscopic masses and uncertainties, there are a class of spacetime coarse grainings extending over a time $T \ll T_{\text{dynamical}} \ll T_{\text{decoherence}}$ to which instantaneous alternatives are an excellent approximation both with respect to decoherence and with respect to probabilities. For systems of small mass, these results suggest that the regime of validity of such approximations may be more limited. In particular, in realistic measurement situations of light systems which extend over time, it may be necessary to take the details of the experimental arrangement into account, so that the alternatives describing the outcome of the measurement refer to the alternative configurations of the apparatus rather than the system being measured if they are to be well approximated by instantaneous alternatives. It would be desirable to have more detailed and realistic models to confirm this.

Acknowledgments

The authors are grateful to David Craig for much advice on the evaluation of the integrals in this paper. This work was supported under NSF grants PHY94-07065 and PHY94-07194. The work of R.M. was supported by a UCSB College of Creative Studies Summer Undergraduate Research Fellowship.

References

- [1] R.P. Feynman, Rev. Mod. Phys., 20, 267, 1948.
- [2] M.B. Mensky, Phys. Rev. D, 20, 384, 1979; Theor. Math. Phys., 75, 357, 1988.
- [3] C. Caves, Phys. Rev. D, 33, 1643, 1986; *ibid.*, 35, 1815, 1987.
- [4] A. Schmid, Ann. Phys., 173, 103, 1987.
- [5] J.B. Hartle, Phys. Rev. D, 38, 2985, 1988.
- [6] N. Yamada and S. Takagi, Prog. Theor. Phys., 85, 985, 1991.
- [7] R. Sorkin, Int. Jour. Theor. Phys., 33, 523, 1994.
- [8] D. Marolf, Phys. Rev. A, 50, 939, 1994
- [9] J.B. Hartle, *The Quantum Mechanics of Cosmology*, in *Quantum Cosmology and Baby Universes: Proceedings of the 1989 Jerusalem Winter School for Theoretical Physics*, ed. by S. Coleman, J.B. Hartle, T. Piran, and S. Weinberg, World Scientific, Singapore (1991) pp. 65-157.
- [10] J. B. Hartle, Phys. Rev. D **44**, 3173 (1991)
- [11] N. Yamada and S. Takagi, Prog. Theor. Phys., 86, 599, 1991; *ibid.*, 87, 77, 1992.
- [12] I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series, and Products*, (Academic Press, New York, 1965)